

# Laser Speckle Simulation in Rotationally Symmetric Triangulation Sensor

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**Abstract** Speckle is the fundamental uncertainty factor in laser triangulation. A method to simulate the speckle in rotationally symmetric triangulation was presented and the simulated image was obtained. In this kind of triangulation sensors, the incident laser point will be imaged to a ring on the detector and the speckle is accordingly arc shaped. Properties of this kind of speckle were studied. The speckle size in radius direction of the ring obeyed the subjective speckle, and is determined by the number aperture of the optical system. In tangent direction of the ring, the speckle is essentially an objective speckle, its size is determined by the optical path length from the object to the detector, the area of the incident laser spot, as well as the radius of the imaged ring because of optical path was folded. Experiments showed that the simulation result was coincident with speckle theory. Based on the simulation, an analysis of the uncertainty limits of rotationally symmetric triangulation sensor was given. It shows that using the optical layout in our sensor, an uncertainty about 1/5 of traditional triangulation was estimated with same optical system numerical aperture and grey centroid algorithm.

**Keywords** optical measurement, laser triangulation sensor, speckle, simulation, rotationally symmetric

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## 旋转对称三角传感器激光散斑的仿真研究

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**摘要** 散斑是激光三角传感器测量不确定度极限的根本影响因素。提出了一种用于旋转对称激光三角传感器的激光散斑的仿真方法, 获得了仿真散斑图像。在旋转对称的三角传感器中, 投射的激光点在检测器上被成像为一个环, 从而散斑也相应是圆弧形。研究了散斑的特性, 该散斑在环的半径方向上服从主观散斑的特性, 其尺寸由光学系统的数值孔径决定。而在环的切线方向上, 其本质上是客观散斑, 由于光学系统存在折返光路, 其尺寸由物体到检测器的光程、投射的激光光斑尺寸和成像圆环的半径决定。实验结果表明, 仿真结果与散斑理论一致。基于仿真给出了对旋转对称三角传感器位移测量不确定度极限的分析, 结果表明, 使用旋转对称形式的传感器光学布局, 在相同的光学系统数值孔径和使用同样的灰度质心算法的情况下, 可达到传统激光三角测量不确定度的 1/5。

**关键词** 光学测量 激光三角传感器 散斑 仿真 旋转对称

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## 1 Introduction

In quality assurance fast and precise measurements of dimension are often demanded. Optical measurement systems are the most suitable ways, and laser triangulation systems are one of the most important methods in them. Classical triangulation sensors are well established<sup>[1]</sup>, but the measurement result in some cases like gaps or edges depends always on the angular orientation of the sensor.

A rotationally symmetric structure of the triangulation is a solution to overcome these problems. The optical system of rotationally symmetric triangulation (RST) can be realized by both reflecting<sup>[2]</sup> and refracting surfaces<sup>[3]</sup>. An approach with reflecting surfaces was chosen in [2], and an optical design method was published by one of the authors to solve this problem<sup>[2,4]</sup>.

The fundamental measurement uncertainty of laser triangulation comes from laser speckle<sup>[5]</sup>. The laser spot on the rough surface introduced different optical path lengths, resulting in a randomly interference in the image plane. This is the cause of laser speckle<sup>[6]</sup>. Since the usage of laser triangulation, researchers are dealing with laser speckle, because it affected the detection of the position of the laser spot. The uncertainty limit of classical laser triangulation sensor is deduced in [5], [7]. But as authors know, there is no research for properties of laser speckle and how speckle affects the uncertainty limits in rotationally symmetric triangulation sensor.

In this paper, the speckle in our RST sensor was simulated. The statistical properties especially the size of speckle were studied. The results of simulation and experiment were compared and an analysis of the uncertainty limits of rotationally symmetric triangulation sensor was given. The article was arranged as follows. The first section introduced the RST sensor and the basic theory of laser speckle. The second section gave the simulation results of the speckle and some basic analysis of the results. The third section showed the fundamental measurement uncertainty in our sensor and

the last section presented some concluding remarks and following work of our sensor.

## 2 Rotationally symmetric triangulation and laser speckle

The working principle of classical triangulation sensor is as follows: A laser spot is projected to an object. On the surface of the object the spot is scattered in all directions. An optical system images a part of the scattered rays to a detector and makes a spot on the detector. A movement of the laser spot on the object can be determined by measuring the movement of the spot on the detector. It's a fast and easy task for CCD or PSD detectors, but the orientation of the sensor, the shape of the object, and other properties of the object's surface will affect the measurement result. Rotationally symmetric setting up is a good way to resolve these problems. The basic layout of the optical system is shown in Fig. 1. In this layout, the incident laser point will be imaged to a ring. So the displacement of the object will be expressed by the radius of the ring. And the speckles in the ring will be the fundamental uncertainty factor of radius detection.

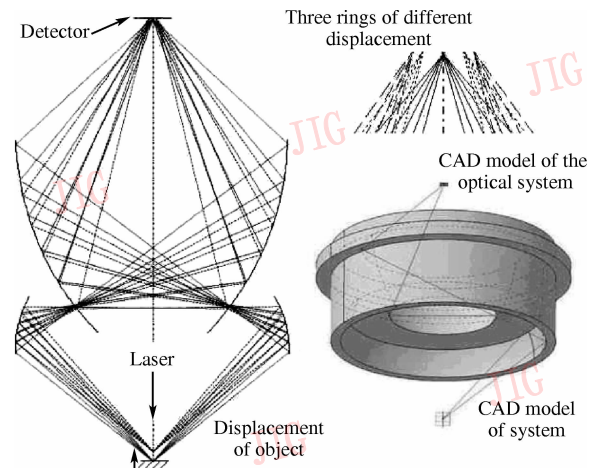


Fig. 1 Basic layout of an optical system with two mirrors

It is well known that when coherent light is incident on a surface which is optical rough, a random intensity pattern is formed. This granular random pattern is known as speckle. For every point in space, the optical lengths from points on a rough surface to it are different and random. This results in a random

interference, and this is called objective speckle. As shown in Fig. 2, when there is an optical system, there will be some difference in the speckle pattern, and this is named subjective speckle.

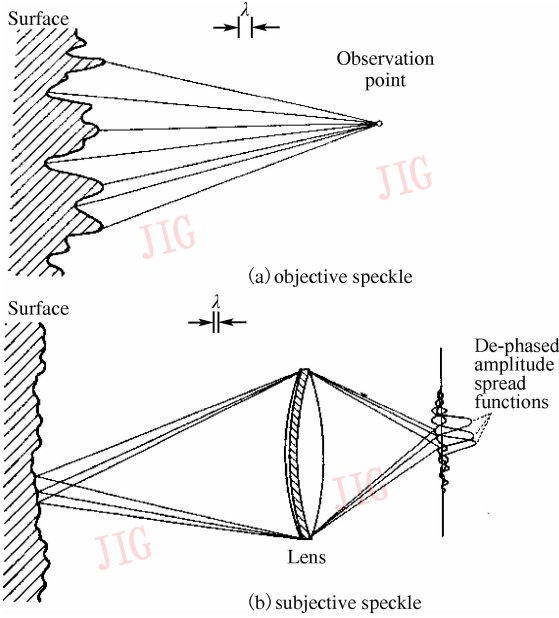


Fig. 2 The principle of two kinds of laser speckle

Because the speckle is essentially a statistical phenomenon, its main properties are presented with some statistical values. The probability distribution of intensity  $I$  and phase  $\theta$  is denoted as:

$$p(I) = \int_{-\infty}^{\infty} \frac{1}{\langle I \rangle} \exp\left(-\frac{\xi}{\langle I \rangle}\right) d\xi = \exp\left(\frac{-I}{\langle I \rangle}\right) \quad (1)$$

$$p_{\theta}(\theta) = \int_{-\pi}^{\pi} p_{I,\theta}(I, \theta) dI = \begin{cases} \frac{1}{2\pi} & -\pi \leq \theta \leq \pi \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where  $\langle I \rangle$  is the average intensity.

The second-order statistics, especially the first zero point of auto-covariance gives out a good estimation for the size of the speckle. The speckle size in free space and imaging system were given as:

$$\delta_R = 0.61 \cdot \lambda L / R \quad (3)$$

$$\delta_R = 0.61 \cdot \lambda / NA \quad (4)$$

where  $\lambda$  is the wavelength of incident laser,  $L$  is the distance from the detector to the surface,  $R$  is the radius of incident laser spot, and  $NA$  is the number aperture of the imaging system.

### 3 Simulation of the speckle in rotationally symmetric traingu-lation sensor

#### 3.1 The simulation and result

As shown in Fig. 1, an ideal point in the incident laser spot will make a ring in the image plane, and as some previous experiments had shown, a little shift of the point on the object surface will introduce a corresponding shift of the ring in image plane. Then, on the assumption that the optical system is a non-aberration one, the e-field of a given point  $E(x, y)$  can be calculated as:

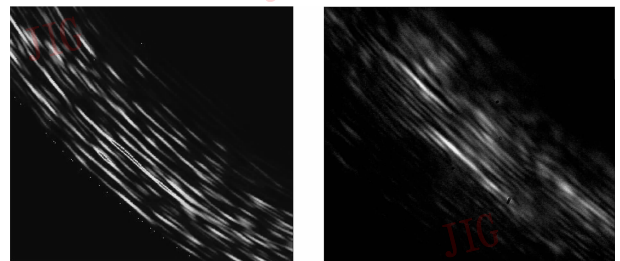
$$E = \sum_{spot} E_k \quad (5)$$

where  $spot$  is all points in the illumination spot, and the surface is rough enough to the wavelength of laser, which means the initial phase distribution of scattered wave is uniform in  $[-\pi, \pi]$ , then every  $E_k$  can be calculated as:

$$E_k = E_0 \frac{NA}{\lambda} \exp\left(i \frac{2\pi}{\lambda} Z_k\right) \text{sinc}\left\{2 \frac{NA}{\lambda} \cdot [R_0 + 0.2\delta - \sqrt{(x - x'_k)^2 + (y - y'_k)^2}]\right\} \quad (6)$$

where  $E_0$  is the amplitude of illumination spot,  $Z_k$  is the optical length (here is the  $\delta$  of the surface), and  $\delta$  is the height of the point in laser spot,  $\lambda$  is wavelength of illumination laser,  $R_0$  is the current radius of the ring,  $(x'_k, y'_k)$  is the centre of displaced ring in image plane.

The simulation result image was shown in Fig. 3. As a comparison, in Fig. 3(b), the real speckle image which is gotten by a microscope in Fig. 4 is also presented.



(a) Speckle pattern from simulation (b) Speckle pattern from microscope

Fig. 3 Speckles in RST

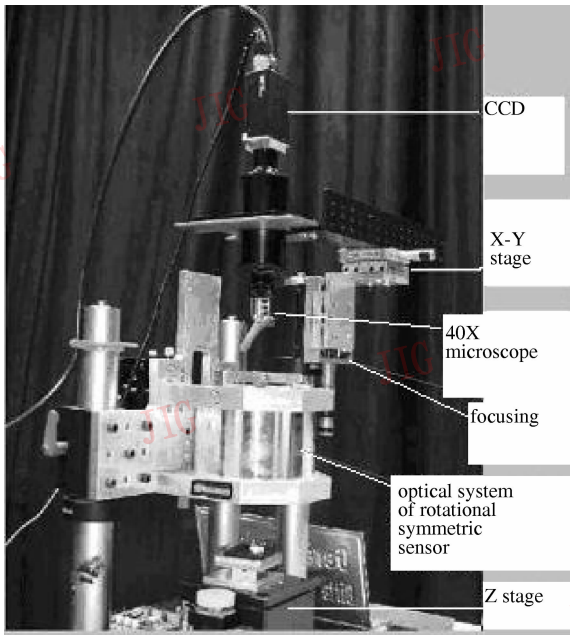


Fig.4 Microscope system to get the real speckle image in the ring

### 3.2 Analysis of the simulation

Some basic parameters of the optical system in the simulation are given in Fig. 5. From which the *NA* of optical system can be determined and is about 0.21. And a laser spot whose radius is  $50\mu\text{m}$  is used in the simulation.

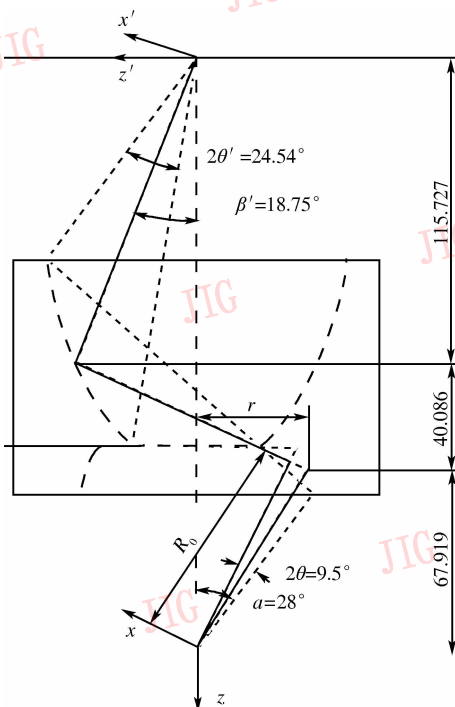
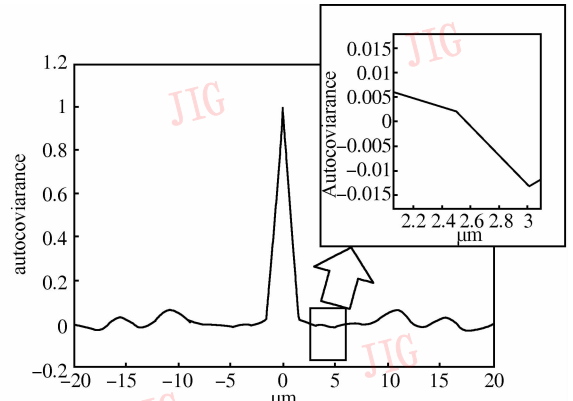
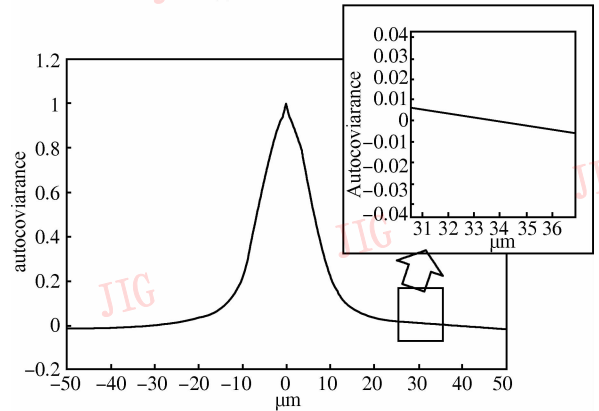


Fig. 5 Basic parameters of the sensor

By calculating the auto-covariance of simulation image in radius and tangent direction, the size of speckle can be estimated, as shown in Fig. 6.



(a) In radius direction



(b) In tangent direction

Fig. 6 The auto-covariance function of the speckle

Fig. 7 gives the first zeros of auto-covariance (speckle size) in radius and tangent directions as well

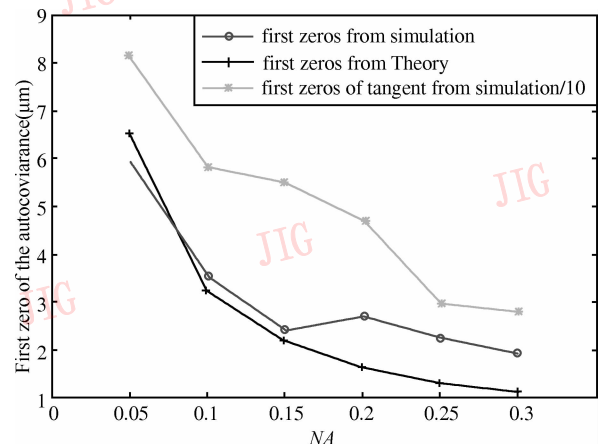


Fig. 7 First zeros of auto-covariance in radius and tangent directions, and theory values in radius direction when *NA* changed from 0.05 to 0.30

as the theoretical value in radius direction when  $NA$  changed from 0.05 to 0.30. Because of overlapping of speckles, it looks that the tangent speckle also changes in the same way of the radius speckle.

Fig. 8 shows the first zeros of auto-covariance (speckle size) with incident laser spot radius change from  $180\mu\text{m}$  to  $420\mu\text{m}$  with step  $60\mu\text{m}$ , it is apparent that tangent speckle get smaller with a bigger spot, a property of objective speckle shown in equal (3). The radius size almost keeps constantly.

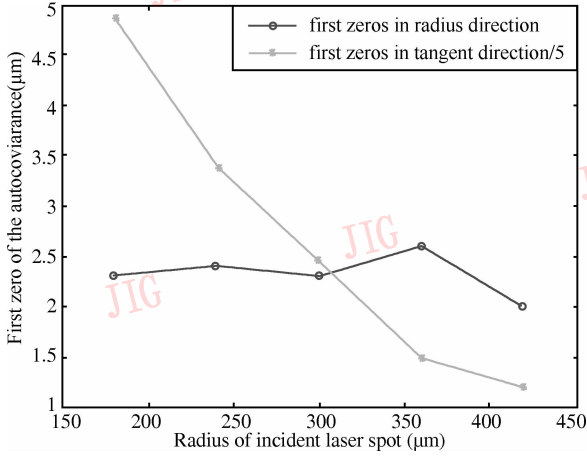


Fig. 8 First zeros of auto-covariance in radius and tangent directions, with the incident laser spot radius change from  $180\mu\text{m}$  to  $420\mu\text{m}$  with step  $60\mu\text{m}$

From these properties, conclusion can be drawn that the speckles in rotationally symmetric laser triangulation have different properties in tangent and radius directions. Because the optical system can be considered as a bended cylinder lens, so speckles in the ring are crosses of a series of speckles in cylinder lenses. From this base, the speckle size in the radius direction is an objective speckle determined by the  $NA$  of the mirror, as in equal (4), and the speckle size in tangent direction is a subjective speckle determined by the size of laser spot and the optical path length. But because the optical path is folded by mirrors, the tangent size of speckle will increase accordingly when the radius size increases.

#### 4 Uncertainty limit with grey centroid algorithm

In order to find the uncertainty limit of

displacement  $Z$ , the deviation of the peak detection in the CCD (image) must be found. Firstly, the ring in the image plane was considered as a series of normal triangulations in each radius direction. Then in the grey centroid algorithm, variance of measured radius of the ring is:

$$\begin{aligned} \text{var}(r_{\text{cog}}) &= \langle (r_{\text{cog}})^2 \rangle - (\langle r_{\text{cog}} \rangle)^2 \\ &= \left\langle \left( \frac{\sum I(r) \cdot r}{\sum I(r)} \right)^2 \right\rangle - \left( \left\langle \frac{\sum I(r) \cdot r}{\sum I(r)} \right\rangle \right)^2 \end{aligned} \quad (7)$$

where  $I(r)$  is the gray of points in radius direction.

We conduct 40 same simulations with different random surfaces, take the standard deviation of grey centroid as detected peak  $\delta_{\text{peak}}$  in one point, it can be expressed as equation (8), and shown in Fig. 9

$$\delta_{\text{peak}} = \sqrt{\text{var}(r_{\text{cog}})} \approx 1.2 (\mu\text{m}) \quad (8)$$

If in each direction the  $\delta_{\text{peak}}$  is independent, given  $k$  directions, the mean (exception) will have standard deviation  $\bar{\delta}_{\text{peak}} = \frac{1}{\sqrt{k}} \delta_{\text{peak}}$ , but it is not the case because of the tangent speckle.

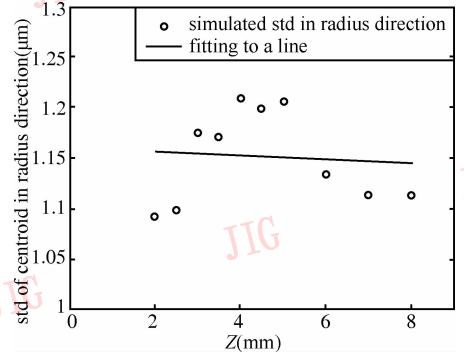


Fig. 9 Standard deviations of grey centroid in radius direction, with displacement  $Z$  from 2mm to 8mm

In one ring, peaks of speckles are certainly somewhat dependent. From the simulation, there are about 150 speckles in one ring circumference, then:

$$\bar{\delta}_{\text{peak}} = \frac{1}{\sqrt{k}} \delta_{\text{peak}} \approx \frac{1.2}{\sqrt{150}} = 0.098 (\mu\text{m}) \quad (9)$$

Because the magnification of the optical system  $M$  is about 0.2, so the uncertainty limit of the sensor is about:

$$\bar{\delta}_z = \frac{\bar{\delta}_{\text{peak}}}{M} = 0.49 (\mu\text{m}) \quad (10)$$

As a comparison, the uncertainty limit in

traditional triangulation with optical parameters presented in this article can be derived as [5]:

$$\bar{\delta}_{zr} = \frac{1}{M} * \frac{\lambda}{2\pi \sin(\theta'/2)} = 2.46(\mu\text{m}) \quad (11)$$

where  $M = 0.2$  is the optical magnification. It is about 5 times uncertainty in rotationally sensor.

It is apparent that in order to get less uncertainty in the sensor, a good way is to increase the  $k$  or decrease  $\bar{\delta}_{\text{peak}}$  in equal (9), which can be gotten by using a lager  $NA$  of the optical system or a bigger entrance pupil.

## 5 Conclusion

In this paper, a simulation of laser speckle in rotationally symmetric triangulation sensor was presented. The pattern obtained from microscope and simulation appeared an arc shape. Properties of this speckle was studied, more attention was payed to the second order statistic properties, such as auto-covariance which means size of it. It was considered that the speckle size in radius direction of the ring obeyed the normal subjective speckle, and is determined by the number aperture of the optical system, while in tangent direction of the ring, the speckle is essentially an objective speckle, its size is determined by the distance from the object to the entrance pupil, the area of the incident laser spot, and the radius of the imaged ring. But because of the folded optical path, it also changes linearly with the radius size.

It was shown that the speckle size, both radius

and tangent will affect the measurement uncertainty limits. Using the optical layout in our sensor, an uncertainty about 1/5 of traditional triangulation was estimated with a grey centroid algorithm. In order to get less uncertainty limit, an optical system with lager  $NA$  should be used.

Some following work includes more precise simulation using ray tracing, and use other algorithms of peak detection to obtain a better measurement uncertainty.

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