Journal of Image and Graphics

中国图象图形学报

ISSN1006-8961
CN11-3758/TB
2012 10
Vol.17 No.

中国科学院遥感应用研究所
中国图象图形学学会 主办
北京应用物理与计算数学研究所

中国精品科技期刊

中国图象图形学报

Zhongguo Tuxiang Tuxing Xuebao

2012 年 10 月　第 17 卷　第 10 期(总第 198 期)

目　　次

中国图象图形学报

刊名题字：宋 健　　月刊(1996 年创刊)　　第 17 卷 第 10 期　　2012 年 10 月 16 日出版

主管单位 中国科学院
主　　办 中国科学院遥感应用研究所
中国图象图形学学会
北京应用物理与计算数学研究所
主　　编 李小文
编辑出版 《中国图象图形学报》编辑出版委员会
北京 9718 信箱 邮编 100101
电子信箱:jig@ irsa. ac. cn
电话:010-64807995 010-82614429
网 址:www. cjig. cn
印刷装订 北京北林印刷厂
广告经营许可证 京朝工商广字第 0346 号
总 发 行 北京报刊发行局
订　　购 全国各地邮局
国外发行 中国国际图书贸易总公司
(中国国际书店)
(北京 399 信箱 邮编 100044)

Superintended by Chinese Academy of Sciences
Sponsored by Institute of Remote Sensing Application, CAS China Society of Image and Graphics Institute of Applied Physics and Computational Mathematics
Chief editor LI Xiaowen
Editor, Publisher Editorial and Publishing Board of Journal of Image and Graphics (P. O. Box 9718, Beijing 100101, China) E-mail: jig@ irsa. ac. cn
Distributed by Beijing Bureau for Distribution of Newspapers and Journals
Domestic All Local Post Offices in China
Foreign China International Book Trading Corporation (P. O. Box 399, Beijing 100044, China)
Printed by Beijing Beilin Printing House

ISSN 1006-8961　CN11-3758/TB　CODE ZTTXFZ　国内邮发代号：82-831　国外发行代号：M1406　国内定价：45.00 元

Journal of Image and Graphics

(Monthly, Started in 1996)

Vol. 17 No. 10 October 2012

Contents

中图法分类号:P23 **文献标识码**:A **文章编号**:1006-8961(2012)10-1319-08
论文引用格式:盛庆红,姬亭,刘微微,王惠南. 对偶四元数线阵遥感影像几何定位[J]. 中国图象图形学报,2012,17(10):1319-1326.

对偶四元数线阵遥感影像几何定位

盛庆红,姬亭,刘微微,王惠南
南京航空航天大学航天学院,南京 210016

摘 要:提出以对偶四元数为数学工具进行线阵CCD(电荷耦合元件)遥感影像几何定位的全新技术方法。利用对偶四元数建立遥感通用传感器严密成像模型,将光线束的位置和姿态统一用对偶四元数表示,通过传感器扫描光线在空间中的螺旋运动,实现像点到其对应地面点物方坐标的变换,从而克服了成像几何参数(外方位元素)之间的强相关性。按照空间刚体变换线性蒙皮混合理论,可以把刚体变换矩阵分解为平移和旋转两个部分,对平移部分进行线性插值,对旋转部分进行球面插值,从而实现线阵CCD遥感影像外方位元素的解算。按照所建立的成像几何模型,利用某地区Geoeye-1遥感影像进行几何定位实验,实验结果表明新算法获得的几何定位精度优于传统算法,能够解决定位参数之间的相关性问题。

关键词:几何定位;对偶四元数;线阵CCD影像;成像几何模型;空间后方交会

Geo-positioning line-array CCD images with dual quaternion

Sheng Qinghong, Ji Ting, Liu Weiwei, Wang Huinan
College of Astronautics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, *China*

Abstract: In this paper, we propose a new method that uses dual quaternion as the mathematical tool for geo-positioning remote sensing images of linear-array CCD. Dual quaternion is used to establish an universal imaging model of remote sensors and to define the position and attitude of the light beam. In order to overcome the strong correlation between the imaging geometry parameters (exterior orientation), we scan the spiral movement of the sensor light and achieve the transformation between image points and ground points. According to the theory of linear skinning blending of the rigid body transformation, we decompose the rigid body transformation matrix into two parts: translation and rotation. In order to calculate the exterior orientation elements of the remote sensing images of linear-array CCD, we use linear interpolation to the translation part and spherical interpolation to the rotation part. According to the established imaging model, geo-positioning experiments are made with Geoeye-1 remote sensing images. The results show that the new algorithm can obtain better geo-positioning accuracy than traditional algorithms, and succeeds in solving the correlation problem between the positioning parameters.

Key words: geo-positioning; dual quaternion; line-array CCD images; imaging model; space resection

0 引 言

线阵CCD(电荷耦合元件)遥感影像的空间信息提取是遥感和数字摄影测量学科中的重点研究领域,其首要问题是建立传感器成像几何模型,明确像平面坐标系与地球坐标系之间的数学关系,以便进行影像信息的几何量测、相互比较和复合

收稿日期:2011-10-20;**修回日期**:2012-03-22
基金项目:国家自然科学基金项目(41101441,60974107);南京航空航天大学基本科研业务费专项科研项目(1011-56XZA11048)
第一作者简介:盛庆红(1978—),女,讲师,2008年于武汉大学获摄影测量与遥感专业博士学位,主要研究方向为航天摄影测量和遥感影像几何定位。E-mail:qhsheng@nuaa.edu.cn

分析[1]。线阵CCD影像推扫式的成像方式决定了各扫描行的外方位元素随时间变化,而且成像光束窄和基高比小,从而导致共线方程的12个定位参数之间存在很强的相关性,使得误差方程系数矩阵之间存在近似的线性关系,即回归分析中所谓的复共线性[2]。由于最小二乘估计法对于外方位元素的初始值具有较强的依赖性,再加上复共线性的存在。使得最小二乘法的均方差较大,不再是最优估计。因此如何克服成像几何参数之间的强相关性是建立线阵CCD遥感传感器严密成像几何模型最主要的难点。

为此国内外学者们提出了多种解决方案,如增加虚拟误差方程、合并强相关项和定向参数的线元素和角元素分开求解等[3],都不同程度的提高了外方位元素的解算精度,但并未从根本上解决传统空间后方交会共线方程模型存在的相关性和病态问题。为了更好的解决病态问题,闫利等提出了适用于单个线阵CCD影像空间后方交会的四元数理论,并且使用IKONOS影像进行了验证,实验结果表明新方法的外方位元素解算精度明显优于传统的最小二乘估计、岭估计和广义岭估计[4]。江刚武等人使用球面线性插值方法来解算单个线阵CCD影像外方位元素,实验证明外方位元素的解算精度又得到了进一步的提升[5]。但此类方法仅仅将传感器姿态用四元数来表示,未涉及传感器位置(X_s,Y_s,Z_s)描述问题。实际上,遥感传感器轨道位置和姿态随着时间一起发生变化,其分而治之的处理方式必然引入转动和平移的耦合误差,因此转动和平移的统一考虑不仅可简化成像几何模型,更能够提高影像位姿确定的精度。

根据Charles定理[6],任何一个刚体的一般运动都可以通过绕某个轴的转动和沿该轴的平移实现。对偶四元数可以把刚体运动的转动和平移统一考虑,并且数学表达式直观明了。在计算机辅助设计领域,Rooney[7]认为在空间线变换中对偶四元数最有效,对偶四元数的转换组合等计算在数字上更稳定。在航天工程中,王惠南等人运用对偶四元数构建了编队微小卫星相对位、姿测控统一模型[8]。本文利用对偶四元数几何代数理论对线阵CCD遥感影像进行定位,同时考虑旋转变换和平移变换,从而克服线阵CCD影像外方位元素的相关性。

1 基于对偶四元数的严密成像几何模型

1.1 对偶四元数与线阵CCD遥感影像成像的几何关系分析

线阵CCD推扫式成像传感器逐行以时序方式获取2维影像,定义瞬时像平面坐标系o-xy、本体坐标系S-xyz、轨道坐标系S-XYZ和地球坐标系O-XYZ,上述坐标系之间的关系如图1所示。其中,o-xy以影像上每条扫描线l的主点o为原点,沿着扫描线方向为x轴,垂直于扫描线方向为y轴(卫星运行方向);S-xyz原点在卫星质心S,x、y、z轴分别取卫星的3个主惯量轴;S-XYZ的原点在卫星质心S,Z轴指向地心反向,Y轴在卫星轨道面上指向卫星运动的方向,X轴按照右手规则确定;O-XYZ描述地球上的目标定位,原点为地球质心O,Z轴指向天球北极,在像面上形成一条线影像l,此时,S-xyz、S-XYZ和O-XYZ转动的各角方位元素类比成2个对偶角$\hat{\theta}_i = \theta_i + \varepsilon d_i$,将线方位元素矢类比成$S$-$XYZ$和$O$-$XYZ$的位移矢$\boldsymbol{t}$。

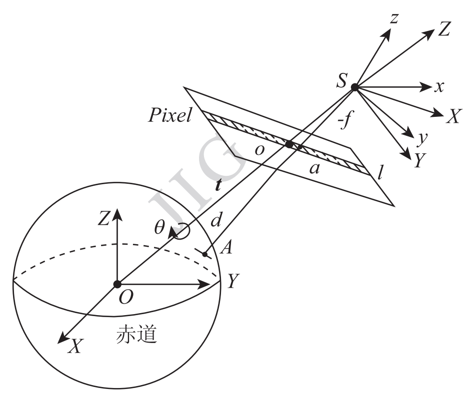


图1 成像几何模型中坐标系之间的对偶四元数描述

Fig. 1 Describe the coordinates of imaging geometrymodel use in dual quaternions

1.2 基于对偶四元数的线阵CCD遥感影像严密成像几何模型

严密成像几何模型描述了在S-xyz和O-XYZ坐标系之间的几何变换关系,利用卫星运动基本矢量和姿态所建立的单线阵推扫式传感器影像坐标与其地面点在O-XYZ下的坐标关系式,则线阵CCD推扫式卫星遥感影像的瞬时严密成像几何模型为[9]

$$x_i = -f\frac{a_{1i}(X-X_{si})+b_{1i}(Y-Y_{si})+c_{1i}(Z-Z_{si})}{a_{3i}(X-X_{si})+b_{3i}(Y-Y_{si})+c_{3i}(Z-Z_{si})}$$

$$y_i = 0 = -f\frac{a_{2i}(X-X_{si})+b_{2i}(Y-Y_{si})+c_{2i}(Z-Z_{si})}{a_{3i}(X-X_{si})+b_{3i}(Y-Y_{si})+c_{3i}(Z-Z_{si})} \tag{1}$$

式中,x_i、y_i 为像点在 o-xy 下的坐标,X、Y、Z 为地面点在 O-XYZ 下的坐标,X_{si}、Y_{si}、Z_{si} 为第 i 扫描行的线方位元素(成像时刻卫星质心在 O-XYZ 中的位置),a_i、b_i、c_i 是第 i 扫描行角方位元素(旋转矩阵 $\boldsymbol{R}_i$ 中的各要素)。设卫星位姿参数的对偶四元数描述为 $\hat{\boldsymbol{q}}_i = [q_{0i},q_{xi},q_{yi},q_{zi}]^{\mathrm{T}} + \varepsilon[r_{0i},r_{xi},r_{yi},r_{zi}]^{\mathrm{T}} = \boldsymbol{q}_i + \varepsilon\boldsymbol{r}_i$,由对偶四元数和线阵 CCD 影像成像几何关系分析可知,角方位元素可用对偶四元数的实部表示,线方位元素需以对偶四元数的实部和对偶部分共同表示,如式(2)所示。通过对旋转矩阵 $\boldsymbol{R}_i$ 和平移矩阵 $\boldsymbol{T}_i = [X_{si},Y_{si},Z_{si}]^{\mathrm{T}}$ 的对偶四元数统一求解,能够同时考虑旋转和平移,从而降低问题处理的复杂度,减小外方位元素之间的强相关性。

$$\boldsymbol{R}_i = \begin{bmatrix} a_{1i} & a_{2i} & a_{3i} \\ b_{1i} & b_{2i} & b_{3i} \\ c_{1i} & c_{2i} & c_{3i} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} q_{0i}^2+q_{xi}^2-q_{yi}^2-q_{zi}^2 & 2(q_{xi}q_{yi}-q_{0i}q_{zi}) & 2(q_{xi}q_{zi}+q_{0i}q_{yi}) \\ 2(q_{yi}q_{xi}+q_{0i}q_{zi}) & q_{0i}^2-q_{xi}^2+q_{yi}^2-q_{zi}^2 & 2(q_{yi}q_{zi}-q_{0i}q_{xi}) \\ 2(q_{zi}q_{xi}-q_{0i}q_{yi}) & 2(q_{zi}q_{yi}+q_{0i}q_{xi}) & q_{0i}^2-q_{xi}^2+q_{yi}^2+q_{zi}^2 \end{bmatrix}$$

$$T_i\begin{bmatrix} X_{si} \\ Y_{si} \\ Z_{si} \end{bmatrix} = \begin{bmatrix} 2q_{0i}r_{xi}-2q_{xi}r_{0i}+2q_{yi}r_{zi}-2q_{zi}r_{yi} \\ 2q_{0i}r_{yi}-2q_{xi}r_{zi}-2q_{yi}r_{0i}+2q_{zi}r_{xi} \\ 2q_{0i}r_{zi}+2q_{xi}r_{yi}-2q_{yi}r_{xi}-2q_{zi}r_{0i} \end{bmatrix} \tag{2}$$

2　基于对偶四元数的线阵 CCD 影像空间后方交会

2.1　外方位元素的线性蒙皮混合算法

空间刚体变换可以使用包含加型变换(平移)和乘型变换(旋转和缩放)的矩阵来表示[10]。为了描述连续的刚体变换,需要进行矩阵的乘法运算以实现矩阵的混合,最终通过单个矩阵描述整个刚性变换过程,目前常使用线性方法来完成矩阵混合[11-12]。线性蒙皮混合算法[13-14]是矩阵线性混合的一种变形,此算法克服了矩阵混合中产生的塌陷问题,保持了整个刚体运动过程中刚性变换的完整性。在进行线性混合时把矩阵分解为平移和旋转两个部分,并对它们分别进行插值。对平移采用线性插值,对旋转采用球面线性插值(SLERP)[15-16]的方法,从而避免了直接矩阵线性混合带来的塌陷问题。根据卫星的运动过程,利用 SLERP 求解每个扫描行的外方位角元素 q_i(旋转部分),使用线性插值法求解外方位线元素 r_i(平移部分)。将一景线阵 CCD 影像的第一扫描行和最后一扫描行的姿态四元数当作外方位姿态的未知数,则任意一扫描行的线元素可采用线性插值求得,即

$$\dot{\boldsymbol{r}}_i = (1-t)\,\dot{\boldsymbol{r}}_1 + t\,\dot{\boldsymbol{r}}_2 \tag{3}$$

任意一扫描行的角元素为

$$\dot{\boldsymbol{q}}_i = C_1(t)\,\dot{\boldsymbol{q}}_1 + C_2(t)\,\dot{\boldsymbol{q}}_2 \tag{4}$$

式中,$\dot{\boldsymbol{r}}_i$ 和 $\dot{\boldsymbol{q}}_i$ 为任意一扫描行的姿态四元数,$\dot{\boldsymbol{r}}_1$、$\dot{\boldsymbol{r}}_2$、$\dot{\boldsymbol{q}}_1$ 和 $\dot{\boldsymbol{q}}_2$ 为影像第一扫描行和最后一扫描行的姿态四元数,$C_1(t)$,$C_2(t)$ 为 $\dot{\boldsymbol{q}}_1$ 和 $\dot{\boldsymbol{q}}_2$ 的函数 $C_1(t) = \dfrac{\sin(1-t)\theta}{\sin\theta}$　$C_2 = \dfrac{\sin t\theta}{\sin\theta}$。

式中

$$\theta = \arccos(\dot{\boldsymbol{q}}_1 \cdot \dot{\boldsymbol{q}}_2) = \arccos(q_{10}q_{20} + q_{1x}q_{2x} + q_{1y}q_{2y} + q_{1z}q_{2z})$$

式(3)矩阵展开为

$$\begin{bmatrix} r_{0i} \\ r_{xi} \\ r_{yi} \\ r_{zi} \end{bmatrix} = \begin{bmatrix} (1-t)r_{01}+tr_{02} \\ (1-t)r_{x1}+tr_{x2} \\ (1-t)r_{y1}+tr_{y2} \\ (1-t)r_{z1}+tr_{z2} \end{bmatrix} \tag{5}$$

式(4)矩阵展开为

$$\begin{bmatrix} q_{0i} \\ q_{xi} \\ q_{yi} \\ q_{zi} \end{bmatrix} = \begin{bmatrix} C_1(t)q_{01}+C_2(t)q_{02} \\ C_1(t)q_{x1}+C_2(t)q_{x2} \\ C_1(t)q_{y1}+C_2(t)q_{y2} \\ C_1(t)q_{z1}+C_2(t)q_{z2} \end{bmatrix} \tag{6}$$

插值参数 t 为

$$t = \frac{i}{n}(0 < t < 1) \tag{7}$$

式中,i 为扫描行行号,n 为整幅影像的扫描行数目。

2.2　基于线性蒙皮混合模型的共线条件方程线性化

在基于对偶四元数线性混合模型的误差方程式

中，未知数包括 r_{01}、r_{x1}、r_{y1}、r_{z1}，r_{02}、r_{x2}、r_{y2}、r_{z2}，q_{01}、q_{x1}、q_{y1}、q_{z1}、q_{02}、q_{x2}、q_{y2}、q_{z2}，则线性化的误差方程式为

$$\begin{cases} v_x = c_{11}\mathrm{d}r_{0i} + c_{12}\mathrm{d}r_{xi} + c_{13}\mathrm{d}r_{yi} + c_{14}\mathrm{d}r_{zi} + \\ \quad c_{15}\mathrm{d}q_{0i} + c_{16}\mathrm{d}q_{xi} + c_{17}\mathrm{d}q_{yi} + c_{18}\mathrm{d}q_{zi} - l_x \\ v_y = c_{21}\mathrm{d}r_{0i} + c_{22}\mathrm{d}r_{xi} + c_{23}\mathrm{d}r_{yi} + c_{24}\mathrm{d}r_{zi} + \\ \quad c_{25}\mathrm{d}q_{0i} + c_{26}\mathrm{d}q_{xi} + c_{27}\mathrm{d}q_{yi} + c_{28}\mathrm{d}q_{zi} - l_y \end{cases} \tag{8}$$

式中

$$c_{11} = 2\frac{f}{Z}(-a_{1i}q_{xi} - b_{1i}q_{yi} - c_{1i}q_{zi}) + 2\frac{x_a}{Z}(-a_{3i}q_{xi} - b_{3i}q_{yi} - c_{3i}q_{zi})$$

$$c_{12} = 2\frac{f}{Z}(a_{1i}q_{0i} + b_{1i}q_{zi} - c_{1i}q_{yi}) + 2\frac{x_a}{Z}(a_{3i}q_{0i} + b_{3i}q_{zi} - c_{3i}q_{yi})$$

$$c_{13} = 2\frac{f}{Z}(-a_{1i}q_{zi} + b_{1i}q_{0i} + c_{1i}q_{xi}) + 2\frac{x_a}{Z}(-a_{3i}q_{zi} + b_{3i}q_{0i} + c_{3i}q_{xi})$$

$$c_{14} = 2\frac{f}{Z}(a_{1i}q_{yi} - b_{1i}q_{xi} + c_{1i}q_{0i}) + 2\frac{x_a}{Z}(a_{3i}q_{yi} - b_{3i}q_{xi} + c_{3i}q_{0i})$$

$$\begin{aligned} c_{15} = -2\frac{f}{Z}(&q_{0i}(X_A - t_0) - q_{zi}(Y_A - t_1) + \\ &q_{yi}(Z_A - t_2) - a_{1i}r_{xi} - b_{1i}r_{yi} - c_{1i}r_{zi}) - \\ 2\frac{x_a}{Z}(&-q_{yi}(X_A - t_0) + q_{xi}(Y_A - t_1) + \\ &q_{0i}(Z_A - t_2) - a_{3i}r_{xi} - b_{3i}r_{yi} - c_{3i}r_{zi}) \end{aligned}$$

$$\begin{aligned} c_{16} = -2\frac{f}{Z}(&q_{xi}(X_A - t_0) + q_{yi}(Y_A - t_1) + \\ &q_{zi}(Z_A - t_2) + a_{1i}r_{0i} + b_{1i}r_{zi} - c_{1i}r_{yi}) - \\ 2\frac{x_a}{Z}(&q_{zi}(X_A - t_0) + q_{0i}(Y_A - t_1) - \\ &q_{xi}(Z_A - t_2) + a_{3i}r_{0i} + b_{3i}r_{zi} - c_{3i}r_{yi}) \end{aligned}$$

$$\begin{aligned} c_{17} = -2\frac{f}{Z}(&-q_{yi}(X_A - t_0) + q_{xi}(Y_A - t_1) + \\ &q_{0i}(Z_A - t_2) - a_{1i}r_{zi} + b_{1i}r_{0i} + c_{1i}r_{xi}) - \\ 2\frac{x_a}{Z}(&-q_{0i}(X_A - t_0) + q_{zi}(Y_A - t_1) - \\ &q_{yi}(Z_A - t_2) - a_{3i}r_{zi} + b_{3i}r_{0i} + c_{3i}r_{xi}) \end{aligned}$$

$$\begin{aligned} c_{18} = -2\frac{f}{Z}(&-q_{zi}(X_A - t_0) - q_{0i}(Y_A - t_1) + \\ &q_{xi}(Z_A - t_2) + a_{1i}r_{yi} - b_{1i}r_{xi} + c_{1i}r_{0i}) - \\ 2\frac{x_a}{Z}(&q_{xi}(X_A - t_0) + q_{yi}(Y_A - t_1) + \\ &q_{zi}(Z_A - t_2) + a_{3i}r_{yi} - b_{3i}r_{xi} + c_{3i}r_{0i}) \end{aligned}$$

$$c_{21} = 2\frac{f}{Z}(-a_{2i}q_{xi} - b_{2i}q_{yi} - c_{2i}q_{zi})$$

$$c_{22} = 2\frac{f}{Z}(a_{2i}q_{0i} + b_{2i}q_{zi} - c_{2i}q_{yi})$$

$$c_{23} = 2\frac{f}{Z}(-a_{2i}q_{zi} + b_{2i}q_{0i} + c_{2i}q_{xi})$$

$$c_{24} = 2\frac{f}{Z}(a_{2i}q_{yi} - b_{2i}q_{xi} + c_{2i}q_{0i})$$

$$\begin{aligned} c_{25} = -2\frac{f}{Z}(&q_{zi}(X_A - t_0) + \\ &q_{0i}(Y_A - t_1) - q_{xi}(Z_A - t_2) - \\ &a_{2i}r_{xi} - b_{2i}r_{yi} - c_{2i}r_{zi}) \end{aligned}$$

$$\begin{aligned} c_{26} = -2\frac{f}{Z}(&q_{yi}(X_A - t_0) - \\ &q_{xi}(Y_A - t_1) - q_{0i}(Z_A - t_2) + \\ &a_{2i}r_{0i} + b_{2i}r_{zi} - c_{2i}r_{yi}) \end{aligned}$$

$$\begin{aligned} c_{27} = -2\frac{f}{Z}(&q_{xi}(X_A - t_0) + \\ &q_{yi}(Y_A - t_1) + q_{zi}(Z_A - t_2) - \\ &a_{2i}r_{zi} + b_{2i}r_{0i} + c_{2i}r_{xi}) \end{aligned}$$

$$\begin{aligned} c_{28} = -2\frac{f}{Z}(&q_{0i}(X_A - t_0) - \\ &q_{zi}(Y_A - t_1) + q_{yi}(Z_A - t_2) + \\ &a_{2i}r_{yi} - b_{2i}r_{xi} + c_{2i}r_{0i}) \end{aligned}$$

$$t_0 = 2r_{xi}q_{0i} - 2r_{0i}q_{xi} + 2r_{zi}q_{yi} - 2r_{yi}q_{zi}$$

$$t_1 = 2r_{yi}q_{0i} - 2r_{zi}q_{xi} - 2r_{0i}q_{yi} + 2r_{xi}q_{zi}$$

$$t_2 = 2r_{zi}q_{0i} + 2r_{yi}q_{xi} - 2r_{xi}q_{yi} - 2r_{0i}q_{zi}$$

$$l_x = x - x', x' = -f\frac{\bar{X}}{\bar{Z}}$$

$$l_y = 0 - y', y' = -f\frac{\bar{Y}}{\bar{Z}}$$

$$\bar{X} = a_{1i}(X - t_0) + b_{1i}(Y - t_1) + c_{1i}(Z - t_2)$$

$$\bar{Y} = a_{2i}(X - t_0) + b_{2i}(Y - t_1) + c_{2i}(Z - t_2)$$

$$\bar{Z} = a_{3i}(X - t_0) + b_{3i}(Y - t_1) + c_{3i}(Z - t_2)$$

根据式(5)(6)求微分，考虑到 r_{0i}，r_{xi}，r_{yi}，r_{zi}，q_{0i}，q_{xi}，q_{yi}，q_{zi} 都是 r_{01}，r_{x1}，r_{y1}，r_{z1}，r_{02}，r_{x2}，r_{y2}，r_{z2}，q_{01}，q_{x1}，q_{y1}，q_{z1}，q_{02}，q_{x2}，q_{y2}，q_{z2}的函数，可得

$$\mathrm{d}q_{mi} = \frac{\partial q_{mi}}{\partial q_{01}}\mathrm{d}q_{01} + \frac{\partial q_{mi}}{\partial q_{x1}}\mathrm{d}q_{x1} + \frac{\partial q_{mi}}{\partial q_{y1}}\mathrm{d}q_{y1} + \frac{\partial q_{mi}}{\partial q_{z1}}\mathrm{d}q_{z1} + \frac{\partial q_{mi}}{\partial q_{02}}\mathrm{d}q_{02} + \frac{\partial q_{mi}}{\partial q_{x2}}\mathrm{d}q_{x2} + \frac{\partial q_{mi}}{\partial q_{y2}}\mathrm{d}q_{y2} + \frac{\partial q_{mi}}{\partial q_{z2}}\mathrm{d}q_{z2}\ (m = 0,x,y,z) \tag{9}$$

$$\mathrm{d}r_{mi} = \frac{\partial r_{mi}}{\partial r_{01}}\mathrm{d}r_{01} + \frac{\partial r_{mi}}{\partial r_{x1}}\mathrm{d}r_{x1} + \frac{\partial r_{mi}}{\partial r_{y1}}\mathrm{d}r_{y1} + \frac{\partial r_{mi}}{\partial r_{z1}}\mathrm{d}r_{z1} + \frac{\partial r_{mi}}{\partial r_{02}}\mathrm{d}r_{02} + \frac{\partial r_{mi}}{\partial r_{x2}}\mathrm{d}r_{x2} + \frac{\partial r_{mi}}{\partial r_{y2}}\mathrm{d}r_{y2} + \frac{\partial r_{mi}}{\partial r_{z2}}\mathrm{d}r_{z2}\ (m = 0,x,y,z) \tag{10}$$

式(9)(10)的 32 个偏导数为

$$\begin{cases} \dfrac{\partial r_{0i}}{\partial r_{01}} = \dfrac{\partial r_{0i}}{\partial r_{x1}} = \dfrac{\partial r_{0i}}{\partial r_{y1}} = \dfrac{\partial r_{0i}}{\partial r_{z1}} = 1 - t \\ \dfrac{\partial r_{0i}}{\partial r_{02}} = \dfrac{\partial r_{0i}}{\partial r_{x2}} = \dfrac{\partial r_{0i}}{\partial r_{y2}} = \dfrac{\partial r_{0i}}{\partial r_{z2}} = t \end{cases} \tag{11}$$

$$\begin{cases} \dfrac{\partial q_{0i}}{\partial q_{01}} = \dfrac{\partial \theta}{\partial q_{01}}\left(\dfrac{\partial C_1(t)}{\partial \theta}q_{01} + \dfrac{\partial C_2(t)}{\partial \theta}q_{02}\right) + C_1(t) \\ \dfrac{\partial q_{0i}}{\partial q_{x1}} = \dfrac{\partial \theta}{\partial q_{x1}}\left(\dfrac{\partial C_1(t)}{\partial \theta}q_{01} + \dfrac{\partial C_2(t)}{\partial \theta}q_{02}\right) \\ \dfrac{\partial q_{0i}}{\partial q_{y1}} = \dfrac{\partial \theta}{\partial q_{y1}}\left(\dfrac{\partial C_1(t)}{\partial \theta}q_{01} + \dfrac{\partial C_2(t)}{\partial \theta}q_{02}\right) \\ \dfrac{\partial q_{0i}}{\partial q_{z1}} = \dfrac{\partial \theta}{\partial q_{z1}}\left(\dfrac{\partial C_1(t)}{\partial \theta}q_{01} + \dfrac{\partial C_2(t)}{\partial \theta}q_{02}\right) \\ \dfrac{\partial q_{0i}}{\partial q_{02}} = \dfrac{\partial \theta}{\partial q_{02}}\left(\dfrac{\partial C_1(t)}{\partial \theta}q_{01} + \dfrac{\partial C_2(t)}{\partial \theta}q_{02}\right) + C_2(t) \\ \dfrac{\partial q_{0i}}{\partial q_{x2}} = \dfrac{\partial \theta}{\partial q_{x2}}\left(\dfrac{\partial C_1(t)}{\partial \theta}q_{01} + \dfrac{\partial C_2(t)}{\partial \theta}q_{02}\right) \\ \dfrac{\partial q_{0i}}{\partial q_{y2}} = \dfrac{\partial \theta}{\partial q_{y2}}\left(\dfrac{\partial C_1(t)}{\partial \theta}q_{01} + \dfrac{\partial C_2(t)}{\partial \theta}q_{02}\right) \\ \dfrac{\partial q_{0i}}{\partial q_{z2}} = \dfrac{\partial \theta}{\partial q_{z2}}\left(\dfrac{\partial C_1(t)}{\partial \theta}q_{01} + \dfrac{\partial C_2(t)}{\partial \theta}q_{02}\right) \end{cases} \tag{12}$$

式中

$$\frac{\partial C_1(t)}{\partial \theta} = ((1-t)\cos\,(1-t)\theta\sin\,\theta - \sin\,(1-t)\theta\cos\,\theta)/\sin^2\theta$$

$$\frac{\partial C_2(t)}{\partial \theta} = \frac{t\cos\,t\theta\sin\,\theta - \sin\,t\theta\cos\,\theta}{\sin^2\theta}$$

$$\frac{\partial \theta}{\partial q_{01}} = -\frac{q_{02}}{\sqrt{1-(\dot{\boldsymbol{q}}_1\dot{\boldsymbol{q}}_2)^2}}$$

$$\frac{\partial \theta}{\partial q_{02}} = -\frac{q_{01}}{\sqrt{1-(\dot{\boldsymbol{q}}_1\dot{\boldsymbol{q}}_2)^2}}$$

$$\frac{\partial \theta}{\partial q_{x1}} = -\frac{q_{x2}}{\sqrt{1-(\dot{\boldsymbol{q}}_1\dot{\boldsymbol{q}}_2)^2}}$$

$$\frac{\partial \theta}{\partial q_{x2}} = -\frac{q_{x1}}{\sqrt{1-(\dot{\boldsymbol{q}}_1\dot{\boldsymbol{q}}_2)^2}}$$

$$\frac{\partial \theta}{\partial q_{y1}} = -\frac{q_{y2}}{\sqrt{1-(\dot{\boldsymbol{q}}_1\dot{\boldsymbol{q}}_2)^2}}$$

$$\frac{\partial \theta}{\partial q_{y2}} = -\frac{q_{y1}}{\sqrt{1-(\dot{\boldsymbol{q}}_1\dot{\boldsymbol{q}}_2)^2}}$$

$$\frac{\partial \theta}{\partial q_{z1}} = -\frac{q_{z2}}{\sqrt{1-(\dot{\boldsymbol{q}}_1\dot{\boldsymbol{q}}_2)^2}}$$

$$\frac{\partial \theta}{\partial q_{z2}} = -\frac{q_{z1}}{\sqrt{1-(\dot{\boldsymbol{q}}_1\dot{\boldsymbol{q}}_2)^2}}$$

将式(11)(12)代入误差方程式(9)(10),得到基于线性蒙皮混合模型的误差方程

$$\begin{cases} v_x = c'_{11}\mathrm{d}r_{01} + c'_{12}\mathrm{d}r_{x1} + c'_{13}\mathrm{d}r_{y1} + c'_{14}\mathrm{d}r_{z1} + c'_{15}\mathrm{d}r_{02} + c'_{16}\mathrm{d}r_{x2} + c'_{17}\mathrm{d}r_{y2} + c'_{18}\mathrm{d}r_{z2} + \\ \quad c'_{19}\mathrm{d}q_{01} + c'_{110}\mathrm{d}q_{x1} + c'_{111}\mathrm{d}q_{y1} + c'_{112}\mathrm{d}q_{z1} + c'_{113}\mathrm{d}q_{02} + c'_{114}\mathrm{d}q_{x2} + c'_{115}\mathrm{d}q_{y2} + c'_{116}\mathrm{d}q_{z2} - l_x \\ v_y = c'_{21}\mathrm{d}r_{01} + c'_{22}\mathrm{d}r_{x1} + c'_{23}\mathrm{d}r_{y1} + c'_{24}\mathrm{d}r_{z1} + c'_{25}\mathrm{d}r_{02} + c'_{26}\mathrm{d}r_{x2} + c'_{27}\mathrm{d}r_{y2} + c'_{28}\mathrm{d}r_{z2} + \\ \quad c'_{29}\mathrm{d}q_{01} + c'_{210}\mathrm{d}q_{x1} + c'_{211}\mathrm{d}q_{y1} + c'_{212}\mathrm{d}q_{z1} + c'_{213}\mathrm{d}q_{02} + c'_{214}\mathrm{d}q_{x2} + c'_{215}\mathrm{d}q_{y2} + c'_{216}\mathrm{d}q_{z2} - l_y \end{cases} \tag{13}$$

式中

$$c'_{11} = s_1c_{11}c'_{12} = s_1c_{12}c'_{13} = s_1c_{13}c'_{14} = s_1c_{14}$$

$$c'_{15} = s_2c_{11}c'_{16} = s_2c_{12}c'_{17} = s_2c_{13}c'_{18} = s_2c_{14}$$

$$c'_{21} = s_1c_{21}c'_{22} = s_1c_{22}c'_{23} = s_1c_{23}c'_{24} = s_1c_{24}$$

$$c'_{25} = s_2c_{21}c'_{26} = s_2c_{22}c'_{27} = s_2c_{23}c'_{28} = s_2c_{24}$$

$$s_1 = 1 - t,\ s_2 = t$$

$$c'_{19} = c_{15}\frac{\partial q_{0i}}{\partial q_{01}} + c_{16}\frac{\partial q_{xi}}{\partial q_{01}} + c_{17}\frac{\partial q_{yi}}{\partial q_{01}} + c_{18}\frac{\partial q_{zi}}{\partial q_{01}}$$

$$c'_{110} = c_{15}\frac{\partial q_{0i}}{\partial q_{x1}} + c_{16}\frac{\partial q_{xi}}{\partial q_{x1}} + c_{17}\frac{\partial q_{yi}}{\partial q_{x1}} + c_{18}\frac{\partial q_{zi}}{\partial q_{x1}}$$

$$c'_{111} = c_{15}\frac{\partial q_{0i}}{\partial q_{y1}} + c_{16}\frac{\partial q_{xi}}{\partial q_{y1}} + c_{17}\frac{\partial q_{yi}}{\partial q_{y1}} + c_{18}\frac{\partial q_{zi}}{\partial q_{y1}}$$

$$c'_{112} = c_{15}\frac{\partial q_{0i}}{\partial q_{z1}} + c_{16}\frac{\partial q_{xi}}{\partial q_{z1}} + c_{17}\frac{\partial q_{yi}}{\partial q_{z1}} + c_{18}\frac{\partial q_{zi}}{\partial q_{z1}}$$

$$c'_{113} = c_{15}\frac{\partial q_{0i}}{\partial q_{02}} + c_{16}\frac{\partial q_{xi}}{\partial q_{02}} + c_{17}\frac{\partial q_{yi}}{\partial q_{02}} + c_{18}\frac{\partial q_{zi}}{\partial q_{02}}$$

$$c'_{114} = c_{15}\frac{\partial q_{0i}}{\partial q_{x2}} + c_{16}\frac{\partial q_{xi}}{\partial q_{x2}} + c_{17}\frac{\partial q_{yi}}{\partial q_{x2}} + c_{18}\frac{\partial q_{zi}}{\partial q_{x2}}$$

$$c'_{115} = c_{15}\frac{\partial q_{0i}}{\partial q_{y2}} + c_{16}\frac{\partial q_{xi}}{\partial q_{y2}} + c_{17}\frac{\partial q_{yi}}{\partial q_{y2}} + c_{18}\frac{\partial q_{zi}}{\partial q_{y2}}$$

$$c'_{116} = c_{15}\frac{\partial q_{0i}}{\partial q_{z2}} + c_{16}\frac{\partial q_{xi}}{\partial q_{z2}} + c_{17}\frac{\partial q_{yi}}{\partial q_{z2}} + c_{18}\frac{\partial q_{zi}}{\partial q_{z2}}$$

$$c'_{29} = c_{25}\frac{\partial q_{0i}}{\partial q_{01}} + c_{26}\frac{\partial q_{xi}}{\partial q_{01}} + c_{27}\frac{\partial q_{yi}}{\partial q_{01}} + c_{28}\frac{\partial q_{zi}}{\partial q_{01}}$$

$$c'_{210} = c_{25}\frac{\partial q_{0i}}{\partial q_{x1}} + c_{26}\frac{\partial q_{xi}}{\partial q_{x1}} + c_{27}\frac{\partial q_{yi}}{\partial q_{x1}} + c_{28}\frac{\partial q_{zi}}{\partial q_{x1}}$$

$$c'_{211} = c_{25}\frac{\partial q_{0i}}{\partial q_{y1}} + c_{26}\frac{\partial q_{xi}}{\partial q_{y1}} + c_{27}\frac{\partial q_{yi}}{\partial q_{y1}} + c_{28}\frac{\partial q_{zi}}{\partial q_{y1}}$$

$$c'_{212} = c_{25}\frac{\partial q_{0i}}{\partial q_{z1}} + c_{26}\frac{\partial q_{xi}}{\partial q_{z1}} + c_{27}\frac{\partial q_{yi}}{\partial q_{z1}} + c_{28}\frac{\partial q_{zi}}{\partial q_{z1}}$$

$$c'_{213} = c_{25}\frac{\partial q_{0i}}{\partial q_{02}} + c_{26}\frac{\partial q_{xi}}{\partial q_{02}} + c_{27}\frac{\partial q_{yi}}{\partial q_{02}} + c_{28}\frac{\partial q_{zi}}{\partial q_{02}}$$

$$c'_{214} = c_{25}\frac{\partial q_{0i}}{\partial q_{x2}} + c_{26}\frac{\partial q_{xi}}{\partial q_{x2}} + c_{27}\frac{\partial q_{yi}}{\partial q_{x2}} + c_{28}\frac{\partial q_{zi}}{\partial q_{x2}}$$

$$c'_{215} = c_{25}\frac{\partial q_{0i}}{\partial q_{y2}} + c_{26}\frac{\partial q_{xi}}{\partial q_{y2}} + c_{27}\frac{\partial q_{yi}}{\partial q_{y2}} + c_{28}\frac{\partial q_{zi}}{\partial q_{y2}}$$

$$c'_{216} = c_{25}\frac{\partial q_{0i}}{\partial q_{z2}} + c_{26}\frac{\partial q_{xi}}{\partial q_{z2}} + c_{27}\frac{\partial q_{yi}}{\partial q_{z2}} + c_{28}\frac{\partial q_{zi}}{\partial q_{z2}}$$

l_x 和 l_y 与式(8)中的相同。

2.3 平差计算

误差方程式(13)的矩阵形式为

$$\boldsymbol{V} = \boldsymbol{C}\,\partial_x + \boldsymbol{l} \tag{14}$$

式中

$$\boldsymbol{V} = [v_x \; v_y]^{\mathrm{T}}, \boldsymbol{l} = [l_x \; l_y]^{\mathrm{T}}$$

$$\boldsymbol{C} = \begin{bmatrix} c'_{11} & c'_{12} & c'_{13} & c'_{14} & c'_{15} & c'_{16} & c'_{17} & c'_{18} & c'_{19} & c'_{110} & c'_{111} & c'_{112} & c'_{113} & c'_{114} & c'_{115} & c'_{116} \\ c'_{21} & c'_{22} & c'_{23} & c'_{24} & c'_{25} & c'_{26} & c'_{27} & c'_{28} & c'_{29} & c'_{210} & c'_{211} & c'_{212} & c'_{213} & c'_{214} & c'_{215} & c'_{216} \end{bmatrix}$$

$$\partial_x = [\mathrm{d}r_{01} \;\; \mathrm{d}r_{x1} \;\; \mathrm{d}r_{y1} \;\; \mathrm{d}r_{z1} \;\; \mathrm{d}r_{02} \;\; \mathrm{d}r_{x2} \;\; \mathrm{d}r_{y2} \;\; \mathrm{d}r_{z2} \;\; \mathrm{d}q_{01} \;\; \mathrm{d}q_{x1} \;\; \mathrm{d}q_{y1} \;\; \mathrm{d}q_{z1} \;\; \mathrm{d}q_{02} \;\; \mathrm{d}q_{x2} \;\; \mathrm{d}q_{y2} \;\; \mathrm{d}q_{z2}]^{\mathrm{T}}$$

由于使用对偶四元数描述卫星遥感影像的位姿,所以误差方程式满足条件

$$\begin{cases} q_{01}r_{01} + q_{x1}r_{x1} + q_{y1}r_{y1} + q_{z1}r_{z1} = 0 \\ q_{02}r_{02} + q_{x2}r_{x2} + q_{y2}r_{y2} + q_{z2}r_{z2} = 0 \\ q_{01}^2 + q_{x1}^2 + q_{y1}^2 + q_{z1}^2 = 1 \\ q_{02}^2 + q_{x2}^2 + q_{y2}^2 + q_{z2}^2 = 1 \end{cases}$$

对这 4 个条件等式进行线性化,可以得到线性化方程式

$$q_{01}\mathrm{d}r_{01} + q_{x1}\mathrm{d}r_{x1} + q_{y1}\mathrm{d}r_{y1} + q_{z1}\mathrm{d}r_{z1} + r_{01}\mathrm{d}q_{01} + r_{x1}\mathrm{d}q_{x1} + r_{y1}\mathrm{d}q_{y1} + r_{z1}\mathrm{d}q_{z1} + w_1 = 0$$

$$w_1 = q_{01}r_{01} + q_{x1}r_{x1} + q_{y1}r_{y1} + q_{z1}r_{z1}q_{02}\mathrm{d}r_{02} + q_{x2}\mathrm{d}r_{x2} + q_{y2}\mathrm{d}r_{y2} + q_{z2}\mathrm{d}r_{z2} + r_{02}\mathrm{d}q_{02} + r_{x2}\mathrm{d}q_{x2} + r_{y2}\mathrm{d}q_{y2} + r_{z2}\mathrm{d}q_{z2} + w_2 = 0$$

$$w_2 = q_{02}r_{02} + q_{x2}r_{x2} + q_{y2}r_{y2} + q_{z2}r_{z2}q_{01}\mathrm{d}q_{01} + q_{x1}\mathrm{d}q_{x1} + q_{y1}\mathrm{d}q_{y1} + q_{z1}\mathrm{d}q_{z1} + w_3 = 0$$

$$w_3 = q_{01}^2 + q_{x1}^2 + q_{y1}^2 + q_{z1}^2 - 1q_{02}\mathrm{d}q_{02} + q_{x2}\mathrm{d}q_{x2} + q_{y2}\mathrm{d}q_{y2} + q_{z2}\mathrm{d}q_{z2} + w_4 = 0$$

$$w_4 = q_{02}^2 + q_{x2}^2 + q_{y2}^2 + q_{z2}^2 - 1$$

以上等式可以写为

$$\boldsymbol{B}_x\partial_x + \boldsymbol{W} = 0 \tag{15}$$

式中

$$\boldsymbol{B}_x = \begin{bmatrix} q_{01} & q_{x1} & q_{y1} & q_{z1} & 0 & 0 & 0 & 0 & r_{01} & r_{x1} & r_{y1} & r_{z1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & q_{02} & q_{x2} & q_{y2} & q_{z2} & 0 & 0 & 0 & 0 & r_{02} & r_{x2} & r_{y2} & r_{z2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_{01} & q_{x1} & q_{y1} & q_{z1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_{02} & q_{x2} & q_{x2} & q_{z2} \end{bmatrix}$$

$$\boldsymbol{W} = [w_1 \;\; w_2 \;\; w_3 \;\; w_4]^{\mathrm{T}}$$

对于 n 个地面点,总的误差方程式为

$$\begin{cases} \boldsymbol{V} = \boldsymbol{C}\partial_x + \boldsymbol{l} \\ \boldsymbol{B}_x\partial_x + \boldsymbol{W} = 0 \end{cases} \tag{16}$$

式中,$\boldsymbol{V}$ 和 $\boldsymbol{l}$ 是 $2n \times 1$ 维矩阵,$\boldsymbol{C}$ 是 $2n \times 16$ 维矩阵,n 是地面控制点的数量。假设观测值的权矩阵为 $\boldsymbol{Q}$,采用联系数的直接解法,得:

$$\boldsymbol{Y} = -\boldsymbol{N}^{-1}\boldsymbol{W}_{\mathrm{Y}} \tag{17}$$

式中

$$\boldsymbol{N} = \begin{bmatrix} \boldsymbol{C}^{\mathrm{T}}\boldsymbol{QC} & \boldsymbol{B}_x^{\mathrm{T}} \\ \boldsymbol{B}_x & 0 \end{bmatrix}$$

$$\boldsymbol{Y} = \begin{bmatrix} \partial_x \\ \boldsymbol{K} \end{bmatrix}, \; \boldsymbol{W}_Y = \begin{bmatrix} \boldsymbol{C}^{\mathrm{T}}\boldsymbol{QL} \\ \boldsymbol{W} \end{bmatrix} \tag{18}$$

当地面控制点的数量 $n \geqslant 6$ 时,先给定卫星遥感影像的位置和姿态元素 ∂_x^0 的初始值(本文为 0),然后解算位置和姿态的最或然改正数,再根据式(17)逐步迭代直到改正数小于规定的限差为止。

2.4　算法概述

完整的算法流程如下：

1）输入基本的数据，包括卫星遥感影像的像点以及相应地面坐标系的控制点。

2）确定外方位元素的初始值∂_x^0。在对偶四元数算法中，对于外方位元素的初始值没有特殊的要求，$q_{x1} = q_{x2} = q_{y1} = q_{y2} = q_{z1} = q_{z2} = 0$，$r_{01} = r_{02} = r_{x1} = r_{x2} = r_{y1} = r_{y2} = r_{z1} = r_{z2} = 0$，$q_{01} = q_{02} = 1$。然后开始迭代。

3）根据式(5)(6)计算每一个地面控制点的外方位元素，同时计算对偶四元数算法的旋转矩阵和平移矩阵。

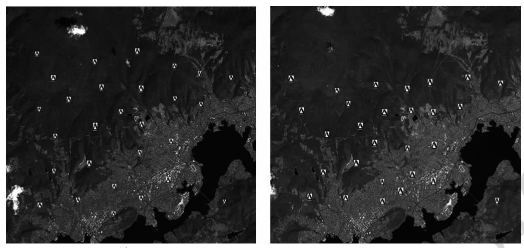
4）根据误差方程式(14)计算矩阵 $\boldsymbol{C}$ 和 $\boldsymbol{l}$，估计最小二乘改正数 ∂_x 的值。

5）根据方程式$\partial_x^1 = \partial_x^0 + \partial_x$更新外方位元素的值，检查 ∂_x 的值是否小于设定的阈值。如果小于阈值，那么迭代结束，∂_x^1就是所估外方位元素的值。否则重复步骤3）和步骤4），直到 ∂_x 小于设定的阈值。

3　实验结果

实验数据选用两幅澳大利亚某地区的 Geoeye-1 影像：影像_01 和影像_02，共同组成了一个立体像对。像元尺寸为 8 μm×8 μm，摄像机的主距为 13.3 m，影像的大小为 26 928×31 668。航高为684 000 m，影像覆盖的地面范围有 50 km×54 km，两幅影像的重叠度超过了 80%。图 2 描述了两幅影像中地面控制点分布的轮廓。对于影像_01 和影像_02，分别选取了 28 个均匀分布的地面控制点，包括 20 个方位点和 8 个检查点。表 1 和表 2 是采用不同的方法对两幅影像进行定位的结果。

在表 1 和表 2 中，V_{xp}、V_{gp}和 V_z 分别为像方平面精度、物方平面精度和高程精度。表 1 和表 2 的结果表明，对 Geoeye-1 遥感影像，广义岭估计、单位四元数和对偶四元数法都能够进行定位处理，而最小二乘估计法解算影像外方位元素不收敛。其中，广义岭估计、单位四元数和对偶四元数法的检查点像



影像_01　　影像_02

图 2　Geoeye-1 影像

Fig. 2　Geoeye-1 images

表 1　不同方法对于影像_01 的定位精度

Table 1　Positional precision of image_01 using different methods

方法	控制点的均方误差			检查点的均方误差		
	V_{xp}/像素	V_{gp}/m	V_z/m	V_{xp}/Pixel	V_{gp}/m	V_z/m
最小二乘估计	不收敛			不收敛		
广义岭估计	3.78	5.54	9.64	2.34	6.44	7.12
四元数(EFP)	2.55	4.13	3.71	1.42	2.84	2.08
四元数(SLERP)	2.57	4.21	3.61	1.63	4.65	3.62
对偶四元数	2.26	2.87	3.39	1.38	1.81	1.83

表 2　不同方法对于影像_02 的定位精度

Table 2　Positional precision of image_02 using different methods

方法	控制点的均方误差			检查点的均方误差		
	V_{xp}/像素	V_{gp}/m	V_z/m	V_{xp}/Pixel	V_{gp}/m	V_z/m
最小二乘估计	不收敛			不收敛		
广义岭估计	4.15	5.54	9.64	2.57	6.44	7.12
四元数(EFP)	2.96	4.13	3.71	1.89	2.84	2.08
四元数(SLERP)	3.01	4.21	3.61	1.95	4.65	3.62
对偶四元数	2.41	2.87	3.39	1.74	1.81	1.83

方平面精度分别为2. 57像素、1. 95像素和1. 74像素,地面点物方平面精度分别为6. 44 m、4. 65 m和1. 81 m,高程精度分别为7. 12 m、3. 62 m和1. 83 m。可见,对偶四元数法定位精度高于广义岭估计和单位四元数法。另外,由于线阵CCD遥感影像的初始和末尾扫面行是用近似值代替的,因此算法存在一定冗余,需要进一步研究。

4 结 论

用对偶四元数统一表示了线阵CCD影像每一扫描时刻的位姿参数,成功构建了线阵CCD影像的严密成像几何模型,克服了成像几何参数之间的相关性。采用线性蒙皮混合算法,保证了遥感卫星运动过程中矩阵混合始终处于刚性变换。实验结果表明,对偶四元数严密成像几何模型能够达到较高的定位精度,并优于传统算法,解决了定位参数之间的相关性问题,在线阵CCD遥感影像对地几何定位中有较好的应用前景。

参考文献(References)

[1] Gong H,Jiang T,Jiang G W,et al. Computing the exterior orientation elements of line-array CCD images by improved two-Step solution method[C]// Proceedings of Congress on Image and Signal Processing. Sanya,China:CISP,2008:702-706.

[2] Liu J. Research of geo-positioning high resolution satellite three-demensional CCD images[D]. Zhengzhou:The PLA Information Engineering University,2003. [刘军. 高分辨率卫星CCD立体影像定位技术研究[D]. 郑州:解放军信息工程大学,2003.]

[3] Zhang Y S,Gong D C,Liu J. Application of High Resolution Satellite Imaging Model、Processing Algorithms and Applied Technology[M]. Beijing:Science Press,2007:17-20[张永生,巩丹超,刘军. 高分辨率遥感卫星应用-成像模型、处理算法及应用技术[M]. 北京:科学出版社,2007:17-20.]

[4] Yan L,Nie Q,Zhao Z. Space resection of line scanner CCD image based on the description of quaternions[J]. Geomatics and Information Science of Wuhan University,2010,35(2):201-204. [闫利,聂倩,赵展. 利用四元数描述线阵CCD影像的空间后方交会[J]. 武汉大学学报·信息科学版,2010,35(2):201-204.]

[5] Jiang G W,Jiang T,Gong H,et al. Exterior orientation of line-array CCD images based on quaternion spherical linear interpolation [C]// Symposium-100 Years ISPRS. Vienna, Austria: ISPRS, 2010:5-7.

[6] Chasles M. Note sur les proprietes generales du sysytem de deux crps semblables entr′eux et places d′une maniere quelcoque dans l′espace et sur le seplacement fini ou infiniment petit d′un solide libre. Ferussac[J]. Bulletin des Sciences Mathhematiques, 1830,14(2):321-326.

[7] Rooney J. A comparison of representations of general spatial screw displacement[J]. Environment and Planning,1978,3(5):45-88.

[8] Ding S W,Wang H N,Liu H Y. Algorithm of vision measure for relative position and pose of rvd spacecrafts based on dual quaternion[J]. Journal of Astronautics,2009,30(6):2145-2150. [丁尚文,王惠南,刘海颖. 基于对偶四元数的航天器交会对接位姿视觉测量算法[J]. 宇航学报,2009,30(6):2145-2150.]

[9] Su W B,Tang X M,Fan D Z,et al. Exterior orientation elements computation of satellite linear array scanner imagery[J]. Science of Surveying and Mapping,2010,35(2):49-50. [苏文博,唐新明,范大昭,等. 线阵CCD卫星影像外方位元素求解的研究[J]. 测绘科学,2010,35(2):49-50.]

[10] Eggert D W, Loruss A, Fisher R B. Estimating 3-D rigid body transformations: a comparison of four major algorithms [J]. Machine Vision and Applications,1997,6(9):272-290.

[11] Alexa M. Linear combination of transformation[J]. ACM Transactions on Graphics,2002,21(3):380-387.

[12] Liu C L. Classifier combination based on confidence transformation[J]. Pattern Recognition,2004,38(4),11-28.

[13] Hejl J. Hardware Skinning with Quaternions [M]//Kirmse A. Game Programming Gems4. Hingham, Mass: Charles River Media,2004:487-495.

[14] Alex M. Building efficient,accurate character skins from examples [J]. ACM Transactions on Graphics,2003,22(3),562-568.

[15] Shoemake K. Animation rotation with quaternion curves [C]// Proceedings of the 12th Annual Conference on Computer Graphics and Interactive Techniques. Boston,USA:ACM,1985:245-254.

[16] Kavan L,Zara J. Spherical blend skinning:a real-time deformation of articulated models[C]//Proceedings of Symposium on Interactive 3D Graphics and Games. New York,USA:ACM,2005:9-16.